

Bulk viscosity of the gluon plasma in a holographic approach

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Abstract

A gravity-scalar model in 5-dim. Riemann space is adjusted to the thermodynamics of SU(3) gauge field theory in the temperature range $1 - 10 T/T_c$ to calculate holographically the bulk viscosity in 4-dim. Minkowski space. Various settings are compared, and it is argued that, upon an adjustment of the scalar potential to reproduce exactly the lattice data within a restricted temperature interval above T_c , rather robust values of the bulk viscosity to entropy density ratio are obtained.

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I. INTRODUCTION

The duality of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in 4-dim. Minkowski space with type IIB superstring theory on $AdS_5 \times S^5$ [1] has initiated a wealth of investigations aimed at exploiting the AdS/CFT correspondence to relate mutually properties of the gravitation sector (which is anti-de Sitter (AdS) in 5-dim. Riemann space) with conformal field theories (CFT). Such techniques look particularly useful for 4-dim. strongly coupled theories, where real-time processes are difficult to access. This in turn applies especially to strongly interacting systems, as subjects to QCD, created in the course of relativistic heavy-ion collisions, i.e. the quark-gluon plasma (QGP). Here, holographic techniques, based on the AdS/CFT correspondence, allow to calculate from suitable gravity duals the wanted observables quantifying properties of the QGP. Among the important quantities is the bulk viscosity which has a potentially strong impact on the analysis of the flow pattern in relativistic heavy-ion collisions [2] and may help to solve the photon- v_2 puzzle [3].

The gravity dual of QCD, even in the pure Yang-Mills sector, is not known. Moreover, QCD is not a CFT since, due to dimensional transmutation, an inherent energy scale is emergent which steers the running coupling. In such a situation and with a lacking top-down approach from string theory, it looks promising to utilize a bottom-up approach which incorporates a selected set of properties one is going to calculate after an appropriate adjustment of the 5-dim. Einstein gravity theory which emerges, strictly speaking, only in the large- N_c limit and at large 't Hooft coupling. A famous example is the gravity-scalar set-up, where a real scalar field is consistently coupled to gravity. The scalar ϕ , dual to an operator \mathcal{O}_ϕ , breaks conformal invariance of AdS space, simulating the corresponding breaking in Yang-Mills theory, the latter being expressed by the trace anomaly relation $T^\mu_\mu = \beta(\alpha)/(8\pi\alpha^2)\text{Tr } F^2$ of the Yang-Mills energy-momentum tensor $T_{\mu\nu}$, β function, running coupling α and trace of the field strength tensor squared $\text{Tr } F^2$. Being interested in thermodynamic properties of the gluon plasma one embeds in the asymptotically AdS space a black brane which introduces a temperature via Hawking temperature and an entropy via Bekenstein-Hawking entropy. Besides the equilibrium thermodynamics, encoded in the gravity metric as dual of the gauge theory energy-momentum tensor, near-to-equilibrium quantities are accessible as correlators based on the energy-momentum tensor. For a medium without conserved charges these are the shear and bulk viscosities as first-order transport coefficients in a gradient expansion.

II. GRAVITY-SCALAR HOLOGRAPHIC MODELS

The class of gravity-scalar duals is defined by the action

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right) + \mathcal{L}_{GH} \quad (1)$$

where \mathcal{L}_{GH} is the Gibbons-Hawking surface term, irrelevant for our purposes, and G_5 denotes the 5-dim. gravity constant. The "potential" $V(\phi)$ determines the self-interaction of the scalar ϕ ; it contains the constant term $V_0 = -12/L^2$ ensuring asymptotic AdS behavior with L being the curvature scale set by the negative cosmological constant. The Riemann space is accordingly specified by extending the conformally flat 4-dim. space-time by the bulk variable u resulting in the ansatz for the infinitesimal line element squared

$$ds^2 = \exp\{2A(u)\} \left(d\vec{x}^2 - f(u)dt^2 + \frac{1}{f(u)}du^2 \right), \quad (2)$$

where (in conformal coordinates) $\lim_{u \rightarrow 0} f(u) = 1$ and $\lim_{u \rightarrow 0} A = \log(L/u)$ ensure the AdS property at the boundary $u \rightarrow 0$ and the simple zero of $f(u_H)$ defines the horizon at $u_H > 0$. The scalar is supposed to have a radial profile $\phi(u)$ which for potentials such as $V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + \dots$ is constrained by the equation of motion to $\phi(u) = \phi_{(4-\Delta)}u^{4-\Delta} + \phi_\Delta u^\Delta + \dots$ near the boundary of AdS, where $\phi_{(4-\Delta)}$ implies an additional term $\propto \int d^4x \phi_{(4-\Delta)} \mathcal{O}_\phi$ as deformation of the original CFT and $\langle \mathcal{O}_\phi \rangle \propto \phi_\Delta$, i.e. ϕ is holographically dual to the operator \mathcal{O}_ϕ with conformal dimension Δ_ϕ . For $\Delta_\phi = 4$, the dual operator is exactly marginal and the scalar field is massless, while for $\Delta_\phi \neq 4$ the source $\phi_{(4-\Delta)}$ introduces a mass scale $\Lambda = \phi_{(4-\Delta)}^{1/(4-\Delta)}$ which explicitly breaks conformal invariance. The mass m and the conformal dimension Δ_ϕ are related by $m^2 L^2 = \Delta_\phi(\Delta_\phi - 4)$, which must satisfy $m^2 L^2 \geq -4$ to fulfill the Breitenlohner-Freedman bound. Renormalizability on the gauge theory side requires $\Delta_\phi \leq 4$, i.e. $m^2 L^2 \leq 0$. While an extension to $1 \leq \Delta_\phi \leq 2$ is possible [4], we restrict our attention to the upper branch of the mass-dimension relation and relevant operators, i.e. $2 < \Delta < 4$. This is already a special setting which follows, e.g., [5, 7, 8] and serves as outline of our analysis below. The improved holographic QCD (IHQCD) model [12], in contrast, is based on different potential asymptotics $V(\phi) - V_0 \propto e^\phi + \dots$ which encodes the running 't Hooft coupling $\lambda \propto e^\phi$ close to the boundary (here at $\phi \rightarrow -\infty$) and results in the marginal case $\Delta_\phi = 4$, while, for large 't Hooft coupling, $V(\phi)$ is constructed to accommodate confinement and a linear glueball spectrum, cf. [5, 9].

III. THERMODYNAMICS

The two basic AdS/CFT thermodynamic relations

$$T = -\frac{1}{4\pi} \frac{df}{du} \Big|_{u_H}, \quad s = \frac{1}{4G_5} \exp\{3A\} \Big|_{u_H} \quad (3)$$

determine the thermodynamics, e.g. by $s(T)/T^3$ for parametrically given temperature $T(u_H)$ and entropy density $s(u_H)$. Here, u_H is the horizon position in the bulk. Einstein's equations determine, via the above conditions at the boundary, the metric coefficients at u_H . To be specific we utilize

$$V(\phi)L^2 = -12 \cosh \gamma \phi + b\phi^2 + \sum_{n=2}^5 c_{2n} \phi^{2n} \quad (4)$$

with $b = 6\gamma^2 + \Delta(\Delta - 4)/2$ from [7, 8], but use solely the matching condition to lattice data of the SU(3) Yang-Mills equation of state in a finite temperature interval above T_c . That is we ignore an *a priori* scale setting at a certain energy and leave thus $2 < \Delta < 4$, γ and c_{2n} as free parameters.

Without further integration constant, the velocity of sound squared, $v_s^2 = d \log s / d \log T$, is given, while the pressure $p = p_c + \int_{T_c}^T dT' s(T')$, energy density $e = -p + sT$ and interaction measure $I = e - 3p$ need one additional constant. A possibility is to employ the lattice input with $p_c = p(T_c)$, which needs a definition of T_c . The IHQCD model has a clear definition of T_c ; other options could be to choose $T_c = T_{min}$, where T_{min} is the minimum of the temperature T as a function of u_H or s/T^3 ; in the latter case, the inflection point T_{ip} can be utilized to define T_c in cases where T as a function of s/T^3 does not have a minimum. If one refrains to catch Yang-Mills features at zero temperature (e.g. a linear glue ball spectrum w.r.t. a radial quantum number) and the latent heat in the deconfinement phase transition as in IHQCD [12] one can adjust the value of T_c arbitrarily; also, G_5 can be chosen without other constraints than the optimum reproduction of a given data set in a restricted temperature interval above T_c . Here, we choose $LT_c = (LT_{min}, LT_{ip})$ and adjust γ , Δ , c_{2n} and G_5/L^3 by minimizing

$$\chi_{s/T^3}^2 = \log \left(\frac{1}{N} \sum_{i=1}^N \left[\sigma(x_i) - y(x_i T_c L) \right]^2 \right), \quad (5)$$

where $\sigma \equiv s(T)/T^3$ refers to the lattice data at N mesh points $x_i \equiv T_i/T_c$ and $y \equiv G_5 s(TL)/(TL)^3$ to the holographically calculated scaled entropy density.

IV. BULK VISCOSITY

The class of gravity-scalar models considered here belongs to so-called two-derivative models which provide the normalized shear viscosity $\eta/s = 1/(4\pi)$, irrespectively of a specific form of $V(\phi)$, at variance with the asymptotic behavior of weakly coupled QCD [13] and the expected minimum near T_c . Higher-order gravity models [10] abandon such a temperature independence. Nevertheless, in the strongly coupled region, $\eta/s = 1/(4\pi)$ represents an intriguingly important result which got popular since the analysis of flow observables in relativistic heavy-ion collisions at RHIC and LHC appeared consistent with that.

The bulk viscosity ζ follows within the present set-up from

$$\frac{\zeta}{\eta} = \left(\frac{d \log V}{d \phi} \right)^2 \left| p_{11} \right|^2 \Big|_{\phi_H} \quad (6)$$

where (using the profile of the scalar field as bulk coordinate) the horizon value of the perturbation p_{11} of the $x_1 x_1$ -metric component is determined by solving a linearized Einstein equation [11].

A. Optimum adjustment to lattice data

As shown in [15], a perfect matching to lattice data is accomplished by the potential (4) for $\Delta = 3.7650$ and $\gamma = 0.6580$ when including the polynomial distortions c_{2n} ; omitting the latter ones (with $\Delta = 3.5976$ and $\gamma = 0.6938$) the match is near-perfect, see left panel in Fig. 1. The bulk to shear viscosity ratio (cf. right panel in Fig. 1) displays a linear section, where $\zeta/\eta = \pi C \Delta v_s^2$ with $C \approx 1.2$, thus fulfilling the Buchel bound $\zeta/\eta \geq 2\Delta v_s^2$ [16]. Such a linear relation $\zeta/\eta \propto \Delta v_s^2 = 1/3 - v_s^2$ is considered in [6] as interesting but as unclear whether it is a generic result of Dp brane gauge theories. With the results of the next subsection we argue that it is generic for the gravity-scalar set-up only for perfect matching to SU(3) Yang-Mills theory. We emphasize that a quasi-particle model [17] obeys quantitatively a similar proportionality in the strong coupling regime, also with the perfect matching of SU(3) Yang-Mills thermodynamics as a prerequisite.

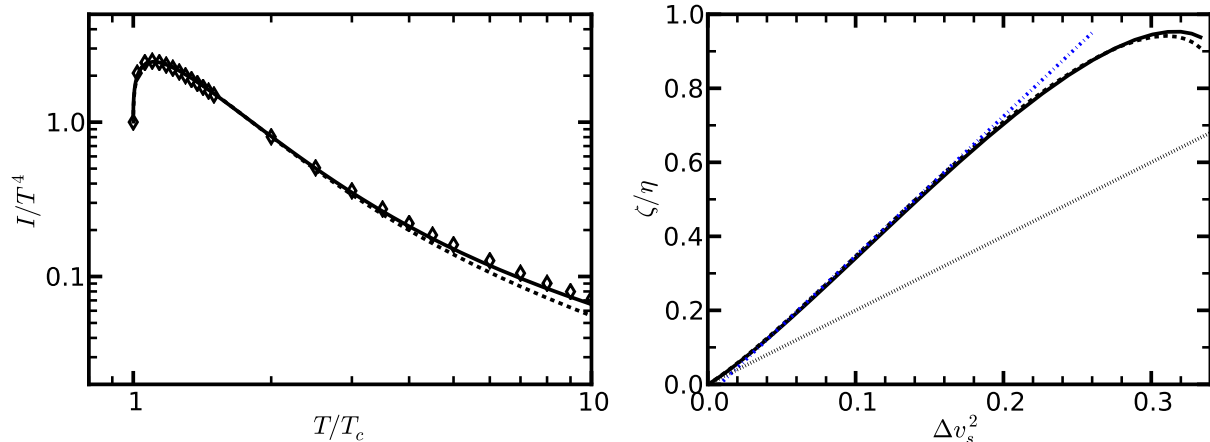


FIG. 1: Left: Scaled interaction measure as a function of T/T_c . The solid (dashed) curve is for the potential (4) with (without) the polynomial distortions $c_{2n}\phi^{2n}$. Other thermodynamic quantities (e.g. v_s^2 , e/T^4 , p/T^4 and s/T^3) agree perfectly (cf. [15]) with the lattice data (symbols, from [14]). Right: Bulk to shear viscosity ratio as a function of the non-conformality measure. The blue dot-dashed line is a linear fit $\zeta/\eta = 1.2\pi\Delta v_s^2 - 0.03$, while the dotted line depicts the Buchel bound $\zeta/\eta = 2\Delta v_s^2$ [16].

B. Dependence of bulk viscosity on potential parameters

We demonstrate now the sensitivity of the bulk viscosity on the parameters of the potential (4) with $c_{2n} = 0$. The analysis is restricted to $3 \leq \Delta \leq 3.9$. The numbers in Fig. 2 indicate selected loci at which we calculate the equation of state and the bulk viscosity exhibited in Fig. 3 below. The deviation measure $\chi_{v_s^2}^2 = \frac{1}{N} \sum_{i=1}^N [v_s^2(x_i) - v_{s,L}^2(x_i T_c L)]^2$ indicates already the (in)accuracy of matching the velocity of sound squared, v_s^2 , from lattice QCD. Hereby, v_s^2 and $v_{s,L}^2$ are obtained from the holographic calculation and the lattice data; x_i and LT_c are as in (5). We emphasize the corridor, in which the points 2, 7 and 12 are localized, which deliver an equally good, though not perfect, reproduction of the lattice data (cf. left column of Fig. 3), due to the individual adjustments of G_5 . The values of ζ/T^3 spread out by a factor of three for $T > T_c$ when comparing the results for all considered loci 1 - 12 (cf. middle column of Fig. 3). In contrast, ζ/η as a function of the non-conformality measure Δv_s^2 looks very much the same for loci 2, 7 and 12, while for the other loci significant variations of ζ/η can be observed, in particular for $\Delta v_s^2 \rightarrow 1/3$, i.e. for $T \rightarrow T_c$. This observation lets us

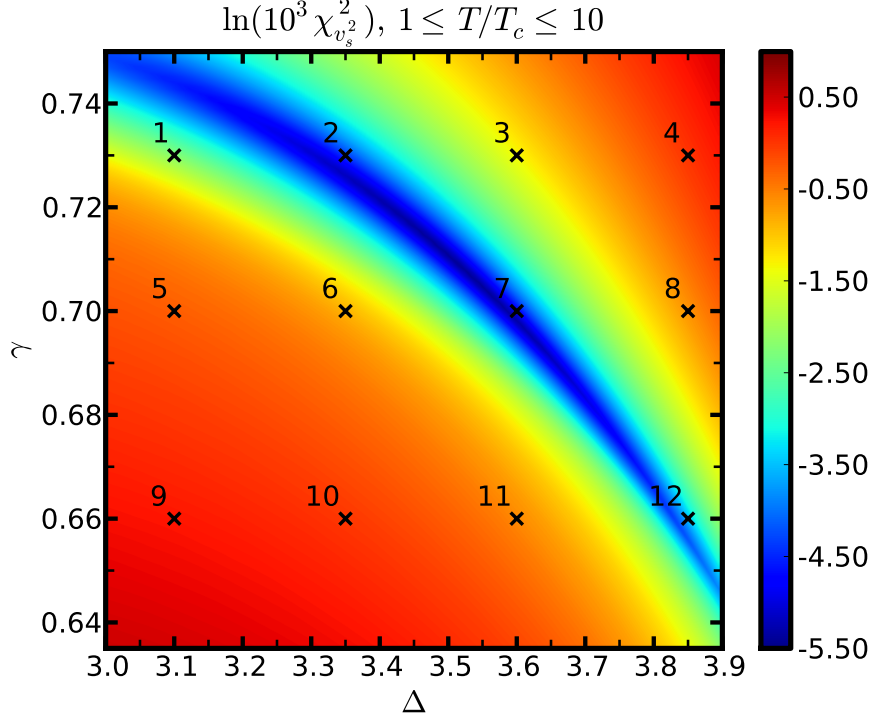


FIG. 2: The $\chi_{v_s^2}^2$ landscape over the γ vs. Δ plane. The numbers indicate loci of selected parameter choices to be analyzed.

argue that a perfect matching of the equation of state may lead to a robust result for ζ/η .

V. SUMMARY

Despite of a lacking gravity dual to thermal SU(3) gauge theory, a gravity-scalar model with an appropriate ansatz for the potential allows for perfectly matching of thermodynamics in the temperature region $(1 - 10)T_c$. Note that no additional constraints are required, e.g. on scale settings or on the confined low-temperature phase or on the asymptotic behavior. The matching condition forces the bulk to shear viscosity ratio to $\zeta/\eta = C\pi\Delta v_s^2$ with $C \approx 1.2$ for $\Delta v_s^2 < 0.25$, in agreement with a previously employed quasi-particle model [17] and the IHQCD model [12]. Without matching, the considered class of potentials exhibits significant variations of both s/T^3 and ζ/η ; deviations from the linear relation $\zeta/\eta \propto \Delta v_s^2$ may occur over a larger range of Δv_s^2 . The increase of ζ/η as a function of the temperature toward T_c , however, seems to be a generic feature. It is always less pronounced than the behavior

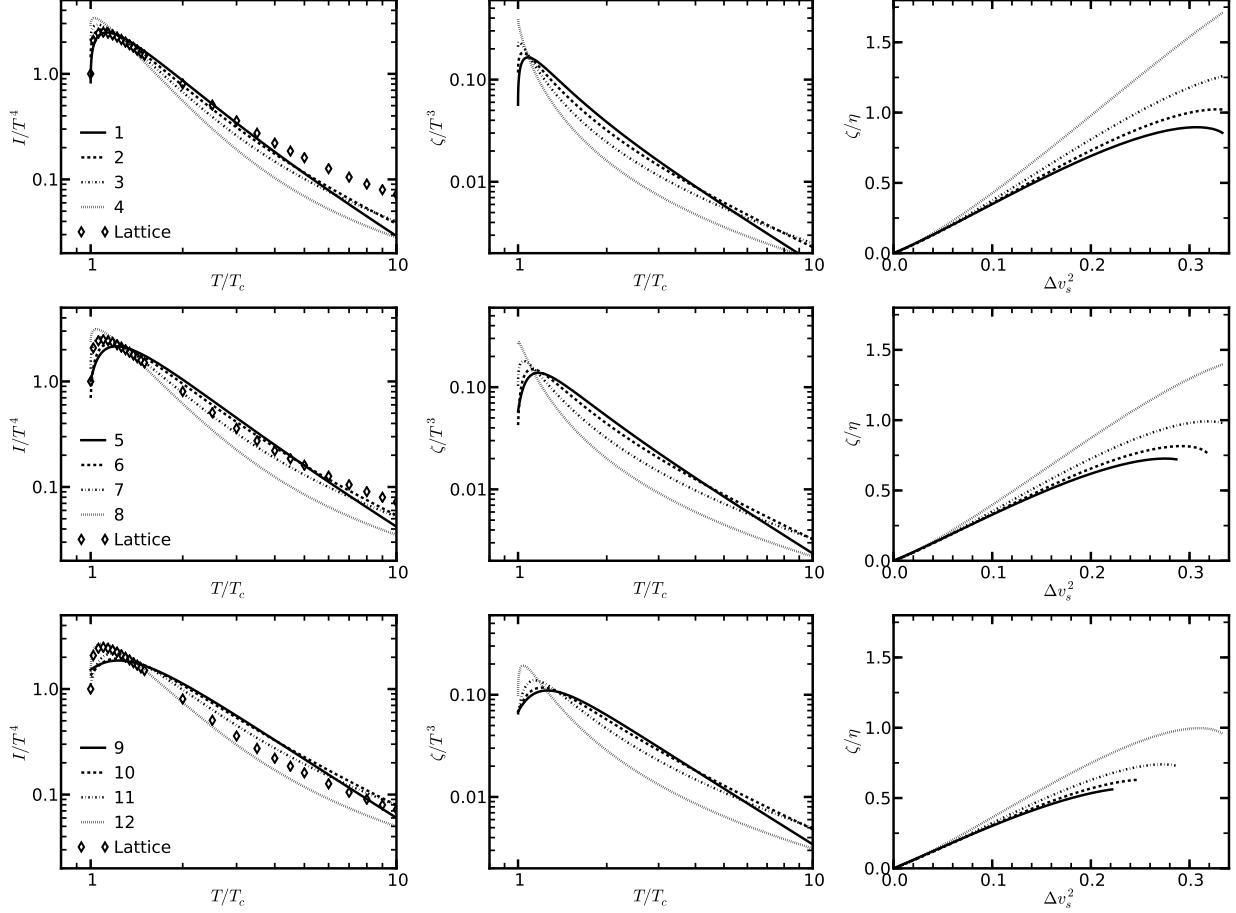


FIG. 3: Equation of state I/T^4 as a function of temperature (left column), scaled bulk viscosity ζ/T^3 as a function of temperature (middle column) and bulk to shear viscosity ratio as a function of non-conformality measure (right column). The numbers in the left panels refer to the loci in the γ vs. Δ plane in Fig. 2.

found in [18].

Our considerations ignore potentially strong curvature effects beyond the classical gravity scenario, the reference to large 't Hooft coupling as well as a direct link to the QCD β function. In so far, we present an exploratory study of a restricted set of observables in a special bottom-up set-up leaving a systematic relation to the *ad hoc* employed AdS/CFT correspondence with controlled deformation to accommodate the non-conformality for further studies.

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- [1] J. M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231
 S. S. Gubser, I. R. Klebanov, A. M. Polyakov, Phys. Lett. B 428 (1998) 105
 E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253
- [2] K. Dusling, T. Schafer, Phys. Rev. C 85 (2012) 044909
 J. Noronha-Hostler, G. S. Denicol, J. Noronha, R. P. G. Andrade, F. Grassi, e-Print:
 arXiv:1305.1981
- [3] G. Basar, D. Kharzeev, V. Skokov, Phys. Rev. Lett. 109 (2012) 202303
- [4] I. R. Klebanov, E. Witten, Nucl. Phys. B 556 (1999) 89
- [5] O. DeWolfe, S. S. Gubser, C. Rosen, D. Teaney, arXiv:1304.7794
- [6] J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal, U. A. Wiedemann, arXiv:1101.0618
- [7] S. S. Gubser, A. Nellore, Phys. Rev. D 78 (2008) 086007
- [8] S. S. Gubser, A. Nellore, S. S. Pufu, F. D. Rocha, Phys. Rev. Lett. 101 (2008) 131601
- [9] F. Nitti, Acta Phys. Polon. Supp. 4 (2011) 661
- [10] S. Cremonini, U. Gürsoy, P. Szepietowski, JHEP 08 (2012) 167
- [11] S. S. Gubser, S. S. Pufu, F. D. Rocha, JHEP 08 (2008) 085
- [12] U. Gürsoy, E. Kiritsis, JHEP 02 (2008) 032
 U. Gürsoy, E. Kiritsis, F. Nitti, JHEP 02 (2008) 019
 U. Gürsoy, E. Kiritsis, L. Mazzanti, F. Nitti, Nucl. Phys. B 820 (2009) 148
- [13] P. B. Arnold, G. D. Moore, L. G. Yaffe, JHEP 0011 (2000) 001
 P. B. Arnold, G. D. Moore, L. G. Yaffe, JHEP 0305 (2003) 051
- [14] Sz. Borsanyi, G. Endrodi, Z. Fodor, S. D. Katz, K. K. Szabo, JHEP 07 (2012) 056
- [15] R. Yaresko, B. Kämpfer, arXiv:1306.0214
- [16] A. Buchel, Phys. Lett. B 663 (2008) 286
- [17] M. Bluhm, B. Kämpfer, K. Redlich, Phys. Lett. B 709 (2012) 77
- [18] F. Karsch, D. Kharzeev, K. Tuchin, Phys. Lett. B 663 (2008) 217